Reg. No. : .....

Name : .....

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B – 3694

Third Semester B.Sc. Degree Examination, December 2016
First Degree Programme under CBCSS
MATHEMATICS
Core Course – 2
MM 1341 – Methods of Algebra and Calculus – 1
(2013 Admission)

Time : 3 Hours

Max. Marks : 80

All the **first 10** questions are **compulsory**. They carry 1 mark **each**.

1. Give an example of a ring homomorphism.
2. Fill in the blank : In $\mathbb{Z}/14\mathbb{Z}$, $[3]^6$ = _____
3. Give an example of a ring.
4. Is $\mathbb{Z}$ under addition, a subgroup of the group of positive real numbers under multiplication.
5. Define exponent of a group.
6. Fill in the blanks : In 3-space, the equation $x^2 + z^2 = 290$ represents a _____
7. Fill in the blanks : If $\vec{u}$ and $\vec{v}$ are nonzero vectors in 3-space, and if θ is the angle between them, then $\cos\theta$ = _____
8. Find the unit vector in the direction of $7\vec{i} + \vec{j} + \vec{k}$.
9. State Kepler's Law of Periods.
10. Find the slope of the line in 2 space that is represented by the vector equation $\vec{r} = (18 - 3t)\hat{i} - (7 - 3t)\hat{j}$.

(10×1=10 Marks)

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Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Give an example of a non abelian group. Justify your answer.

12. If R is a ring with zero element 0, then for any a, b $\in$ R prove that 0a = a0 = 0.

13. Prove that a field has no zero divisors.

14. If R and S are rings and the function f : R $\rightarrow$ S is a homomorphism, then prove that f (–r) = –f(r) for any r in R.

15. State Chinese Remainder Theorem.

16. Find the unit vector in the same direction from the point A(2, 2) to the point B(1, –1).

17. Find the volume of the parallelepiped with adjacent edges $\vec{u} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$.

18. Find parametric equations for the line through the point (3, –4, –1) and parallel to the vector $\vec{i} + \vec{j} + \vec{k}$.

19. Translate the spherical coordinate equation $\rho = 6\cos\phi$ into cylindrical coordinate equation.

20. Find the angle between the planes x + y = 1 and 2x + y – 2z = 2.

21. Find a change of parameter $t = g(\tau)$ for the semicircle $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j}$, $0 \le t \le \pi$. Such that the semicircle is traced counterclockwise as τ increases over the interval [0, 1].

22. If $\vec{r}(t)$ is a vector-valued function in two or three dimensional space and $\|\vec{r}(t)\|$ is constant for all t, then prove that $\vec{r}(t) \cdot \vec{r}'(t) = 0$. **(8×2=16 Marks)**

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Answer **any 6** questions from the questions **23** to **31**. These questions carry **4** marks **each**.

23. Verify Fermat's theorem for the elements of $\mathbb{Z}/7\mathbb{Z}$.

24. Prove that $\mathbb{Z}/m\mathbb{Z}$ is a field if and only if m is prime.

25. If a is any element of a finite group G, then show that the order of a divides the number of elements of G.

26. A sphere S has center in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3 - \sqrt{3}$ units. What is the equation of the sphere ?

27. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors in 3-space, then prove that $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$.

28. Show that the following lines L_1 and L_2 are skew.
$L_1 : x = 1 + 7t,\ y = 3 + t,\ z = 5 - 3t$
$L_1 : x = 4 - t,\ y = 6,\ z = 7 + 2t$

29. Determine whether the vector-valued function $\vec{r}(t) = 3t^2\hat{i} + t^2\hat{j} + t^3\hat{k}$ has continuously turning tangent vector.

30. Find the unit tangent vector at a point t to the curve $\vec{r} = a\cos t\hat{i} + a\sin t\hat{j}$.

31. Sketch the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **(6×4=24 Marks)**

Answer **any 2** questions from among the questions **32** to **35**. These questions carry 15 marks **each**.

32. a) If R is any commutative ring, prove that the set of n-element column vectors for fixed n is an abelian group under addition.

b) Let $G = U_{10}$, the group of units of $\mathbb{Z}/10\mathbb{Z}$. Find the orders of elements of G and the exponent of the group.

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33. a) Find a solution of the following simultaneous congruences :
$x \equiv 3 \pmod{11}$,
$x \equiv 6 \pmod{8}$,
$x \equiv -1 \pmod{15}$
Also find the smallest nonnegative solution.

b) State and prove Cayley's theorem.

34. a) Find the direction cosines of $\vec{v}$, and use them to approximate the direction angles to the nearest degree, where $\vec{v} = \hat{i} + \hat{j} - \hat{k}$.

b) Identify the surface $x^2 - y + z^2 = 0$.

35. a) Find parametric equations of the line tangent to the graph of $\vec{r}(t) = t^2\hat{i} + (2 - \mathit{l}nt)\hat{j}$ at the point $t = 1$.

b) If $\vec{u}$ and $\vec{v}$ are vectors in 3-space, then prove that $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.

(2×15=30 Marks)

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